HYSTERESIS AND BLEACHING OF ABSORPTION BY ELECTRONS ON HELIUM

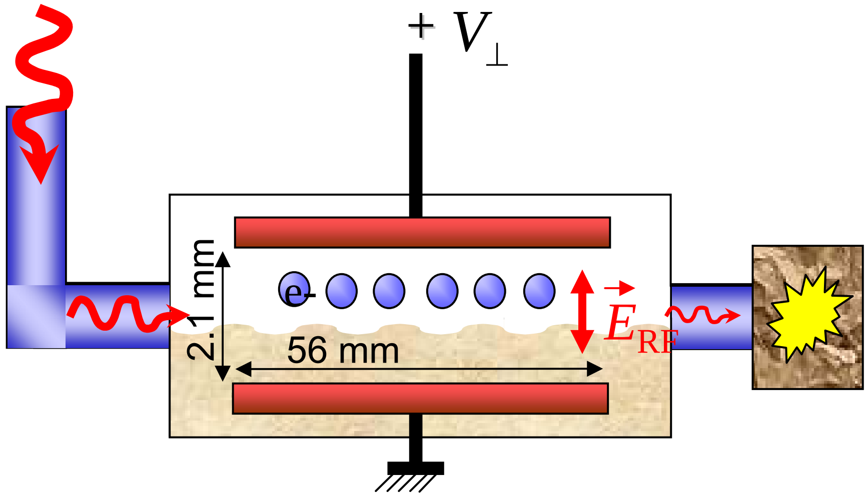
D. Ryvkine,[1] M.J. Lea,[2] and M.I. Dykman[1]

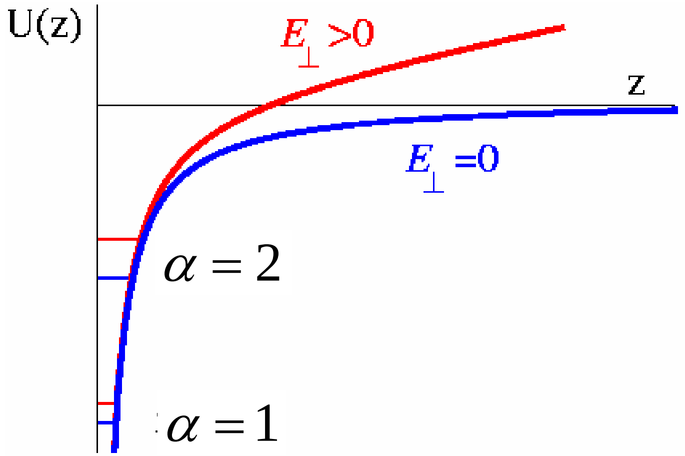
[1] Department of Physics and Astronomy, Michigan State University

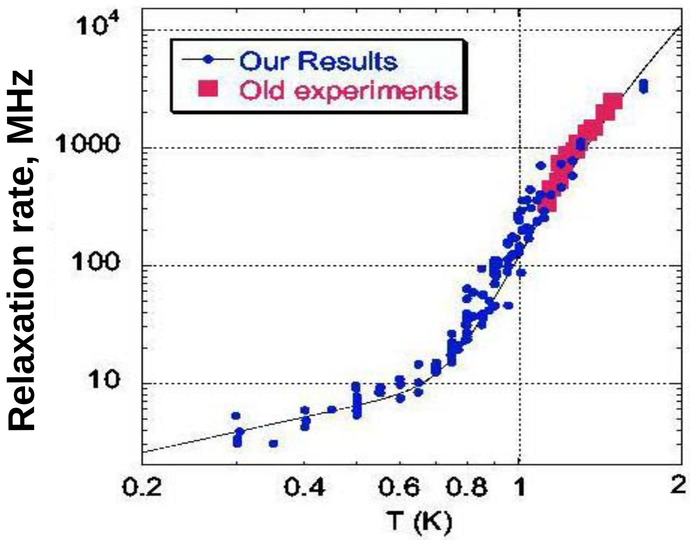
[2] Royal Holloway, University of London

- **Dynamics for slow energy relaxation**
- **Absorption bleaching**
- **Many-electron hysteresis**

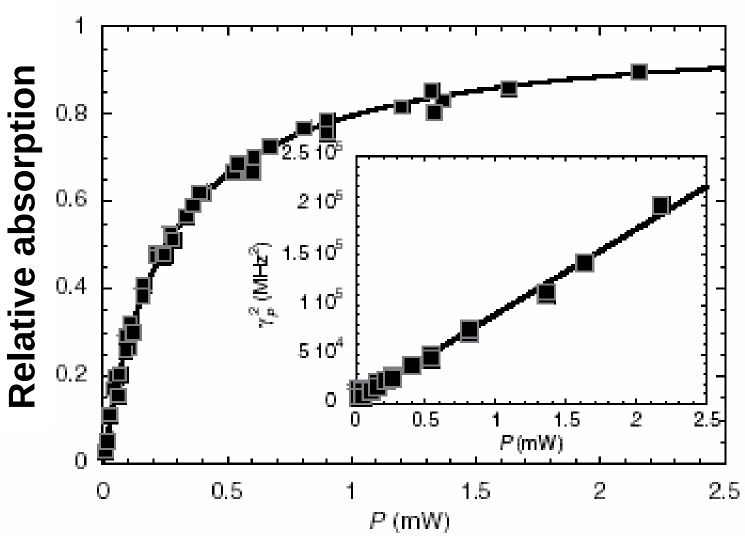
Stark-shift transition frequency by a field $E_\perp$ to tune to 1- 2 resonance





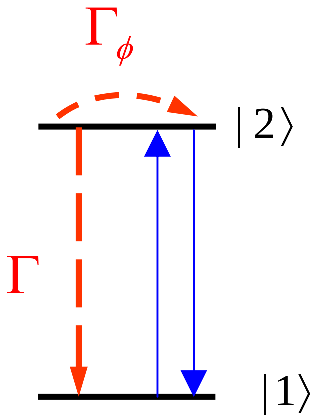


E. Collin et al., PRL (2002)



Interpretation: absorption saturation in a two-level system

Conventional absorption saturation



Nearly resonant driving:

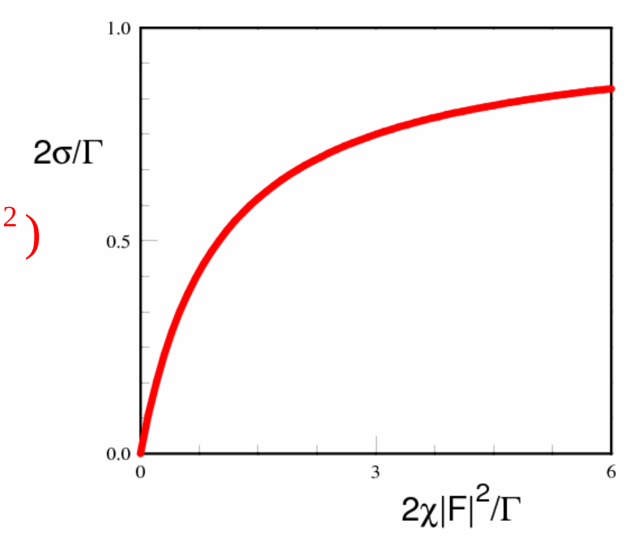
$$H_F = -F\sum_n |2\rangle_{n\,n}\langle 1| \exp(-i\omega_F t) + \text{h.c.} \qquad F = \tfrac{1}{2} e E_{\text{cw}} z_{12}$$

Weak field absorption: $\sigma = \chi\,|F|^2, \quad \chi = \Gamma_0/[\Gamma_0^2 + (\omega_{21} - \omega_F)^2]$

Weak-field linewidth is $\Gamma_0 = \Gamma + \Gamma_\phi$

Power absorbed per unit time is $\sigma\,\omega_F$

Strong field absorption: $\sigma = \Gamma\chi\,|F|^2/(\Gamma + 2\chi\,|F|^2)$



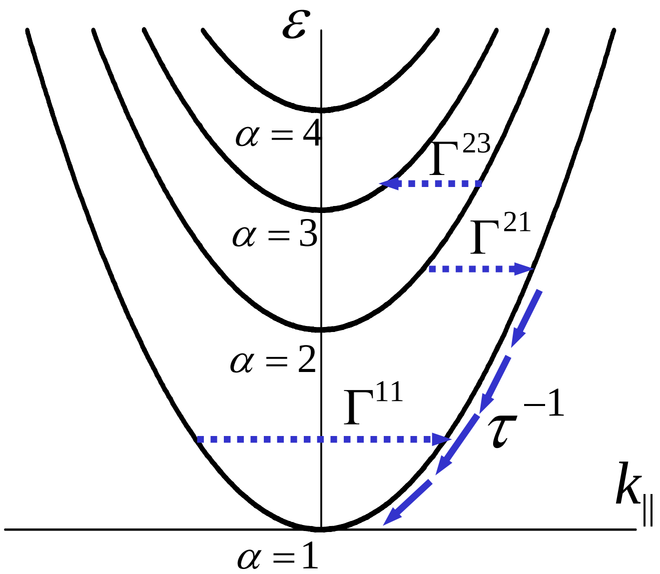
Electrons: bands of in-plane motion instead of energy levels

Electron-electron coupling: thermalization of in-plane motion over time ω_p^{-1},

$$\omega_p = (2\pi e^2 n^{3/2} / m)^{1/2} >> \Gamma^{\alpha\beta}$$

Electron in-plane momentum distribution $\rho^{\alpha\alpha}(\mathbf{p}) \propto \exp(-p^2 / 2mT_e) \quad (k_B = 1)$

$$\rho^{\alpha\beta} = \langle |\beta\rangle_{n\;n}\langle\alpha|\rangle$$



One-ripplon/vapor atoms scattering is quasi-elastic and short-wavelength, $q >> n^{1/2}$

- **intraband scattering,** $\Gamma^{\alpha\alpha}$
- **interband scattering,** $\Gamma^{\alpha\beta} \quad (\alpha \neq \beta)$

$$\Gamma^{\alpha\beta} = 2\pi \sum_{\mathbf{q}} \left| V_{\mathbf{q}}^{\alpha\beta} \right|^2 \left\langle \delta\left(\frac{p^2}{2m} - \frac{(\mathbf{p}+\mathbf{q})^2}{2m} + \varepsilon_\alpha - \varepsilon_\beta \right) \right\rangle \qquad (\hbar = 1)$$

Energy relaxation: two-ripplon/phonon scattering $\tau^{-1} << \Gamma^{\alpha\beta} << \omega_p$

Field Hamiltonian for spatially uniform resonant radiation $F = \frac{1}{2} e E_{\mathrm{cw}} z_{12}$

$$H_F = -F \sum_n |2\rangle_{n\,n}\langle 1| \exp(-i\omega_F t) + \mathrm{h.c.}$$

Frequency detuning is <u>small:</u> $\delta\omega = \omega_F - (\varepsilon_2 - \varepsilon_1), \quad |\delta\omega| << \omega_F$

Short-wavelength scattering, $T_e > \omega_p = (2\pi e^2 n^{3/2} / m)^{1/2}$

$\rightarrow$ **effectively single-electron kinetic equation for a strongly correlated electron system**

In the band index representation

$$\begin{cases} \dot{\rho}^{\alpha\alpha} = -\sum_\beta (\rho^{\alpha\alpha}\Gamma^{\alpha\beta} - \rho^{\beta\beta}\Gamma^{\beta\alpha}) + 2(\delta_{\alpha,1} - \delta_{\alpha,2}) \,\mathrm{Im}(F\rho^{12}) \\ \dot{\rho}^{12} = -(i\,\delta\omega + \Gamma_0)\rho^{12} + iF^*(\rho^{22} - \rho^{11}) \end{cases}$$

T. Ando, 1978

Detailed balance for fast in-plane thermalization $\Gamma^{\alpha\beta} = \Gamma^{\beta\alpha} \exp[(\varepsilon_\alpha - \varepsilon_\beta)/T_e]$

Energy relaxation

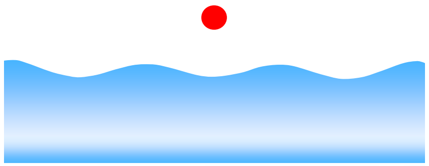
Energy balance equation

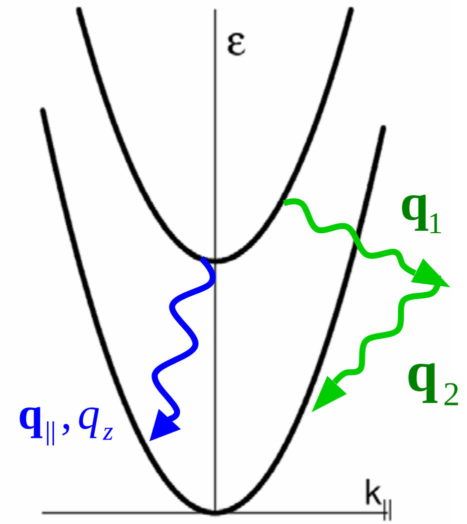
$$\frac{dT_e}{dt} = -\frac{T_e - T}{\tau} + \omega_F \,\mathrm{Im}(F^* \rho^{21})$$

Microscopic mechanisms:

- **Energy diffusion from one-ripplon scattering**
- **Two-ripplon scattering,** $|\mathbf{q}_1 + \mathbf{q}_2| << |\mathbf{q}_{1,2}|$
- **Decay into phonons: modulation of the He dielectric constant,** $q_{\|} << q_z$

$$V(\mathbf{r}) = -\frac{1}{8\pi}\int d\mathbf{r}'\,\delta\varepsilon(\mathbf{r}')E^2(\mathbf{r},\mathbf{r}'), \quad E(\mathbf{r},\mathbf{r}') = e/|\mathbf{r}-\mathbf{r}'|^2$$



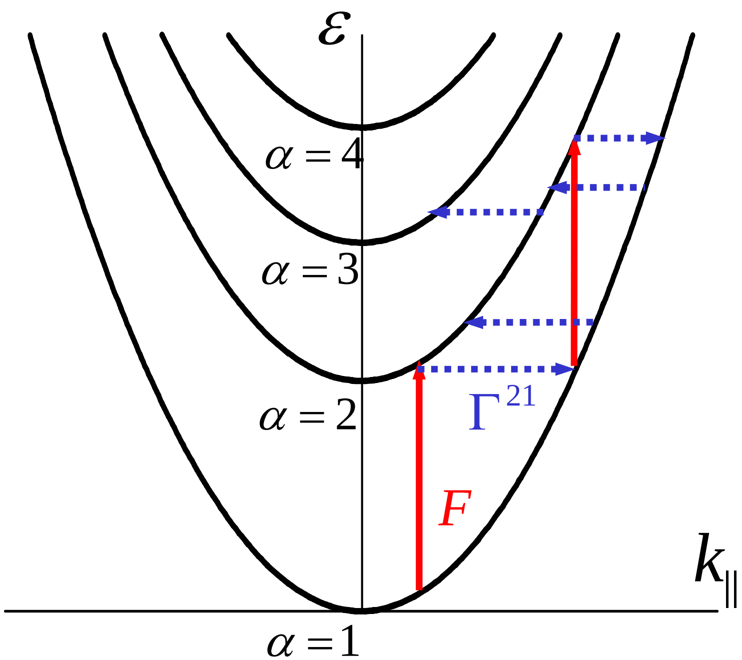


Two-ripplon scattering, kinematic coupling:

$$\tau^{-1} = \tau_0^{-1}(T_e/\hbar\omega_F)^{10/3}, \quad T_e << \hbar\omega_F$$

$$\tau^{-1} \sim \tau_0^{-1}\, T_e/\hbar\omega_F, \qquad T_e \geq \hbar\omega_F$$

Slow energy relaxation, $\Gamma^{21}\tau >> 1$: the rate of field induced transitions does not have to beat the rate of 2→1 relaxation. Excited states are populated and absorption is bleached already for $\chi|F|^2 << \Gamma^{21}$ due to electron heating, $\chi = \Gamma_0 /(\Gamma_0^2 + \delta\omega^2)$



Reminder: absorption saturation for a two-level system requires $\chi|F|^2 > \Gamma^{21}$

$\chi|F|^2 << \Gamma^{21}$ → thermal distribution over bands in the stationary regime

$$\rho^{\alpha\alpha} = Z^{-1}(T_e)\exp(-\varepsilon_\alpha / T_e)$$

$$Z(T_e) = \sum_\alpha \exp(-\varepsilon_\alpha / T_e)$$

Equation for electron temperature

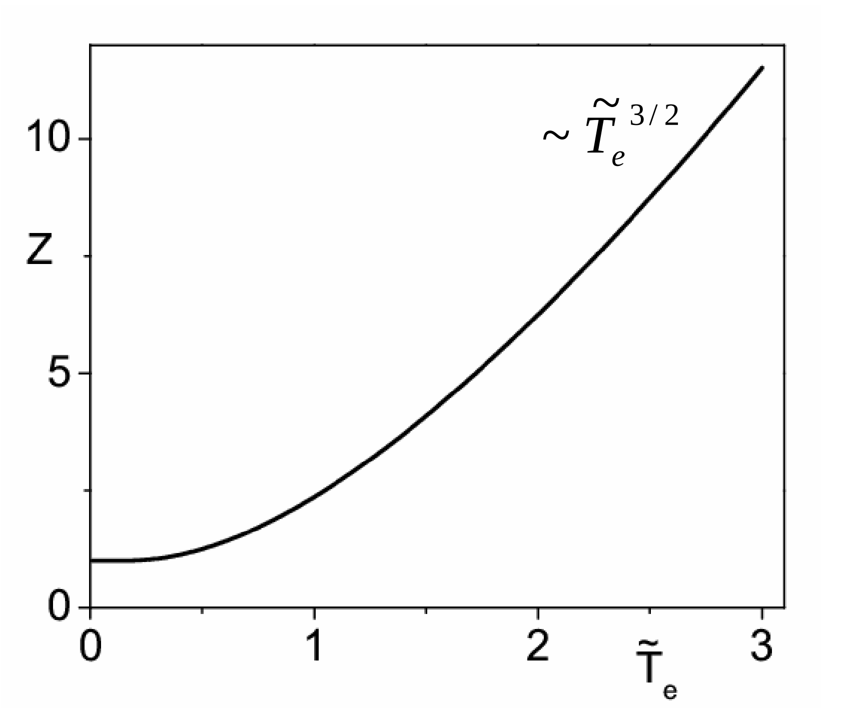
$$\frac{T_e - T}{\tau} = \frac{|F|^2 \chi\omega_F}{Z(T_e)}\left(1 - e^{-\omega_F / T_e}\right)$$

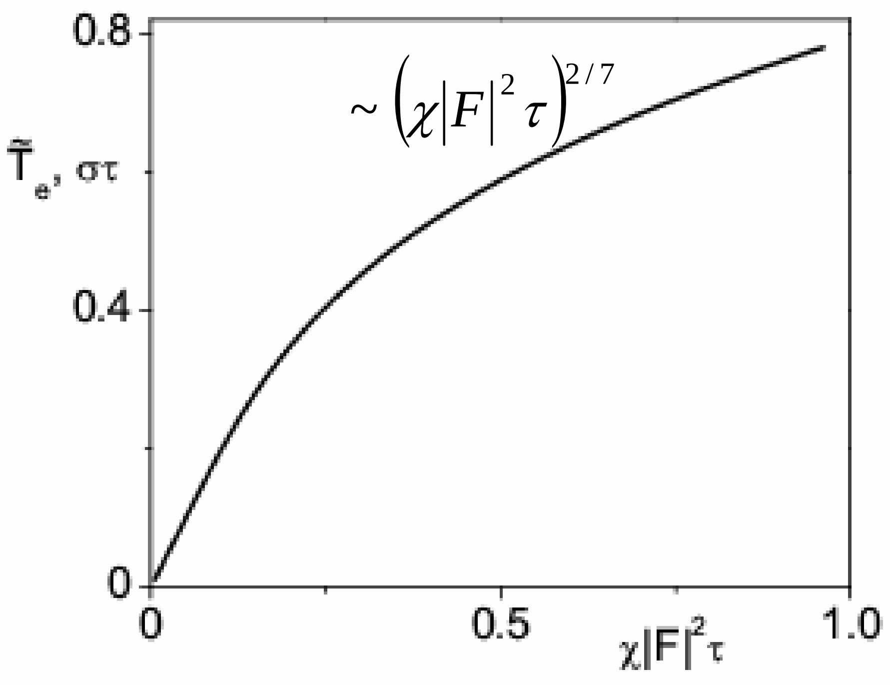
energy is counted off ε_1

Absorption decrease: thermal population of the state $|2\rangle$ and bleaching

$$\sigma = \frac{|F|^2 \chi}{Z(T_e)}\left(1 - e^{-\omega_F/T_e}\right)$$

Constant energy relaxation rate approximation



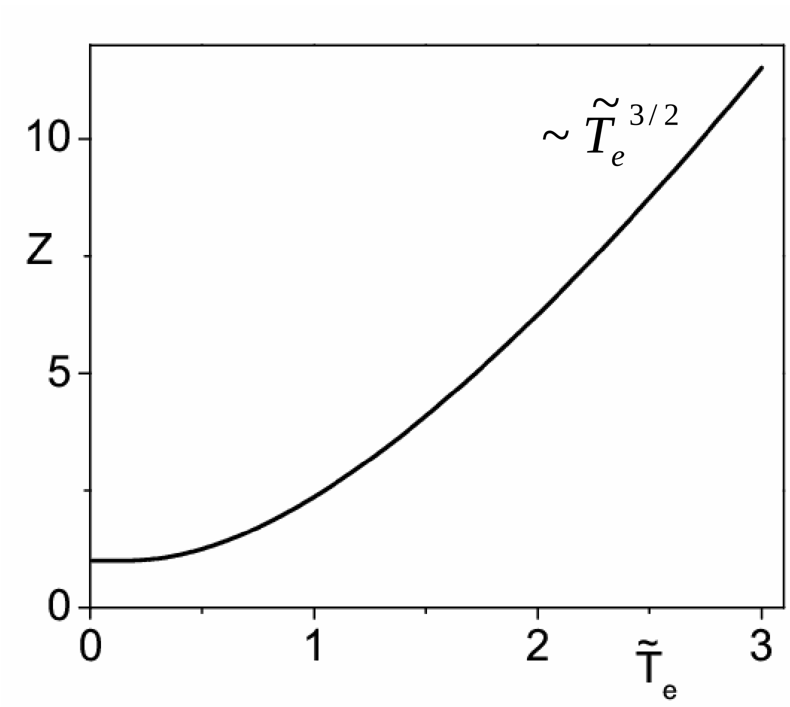


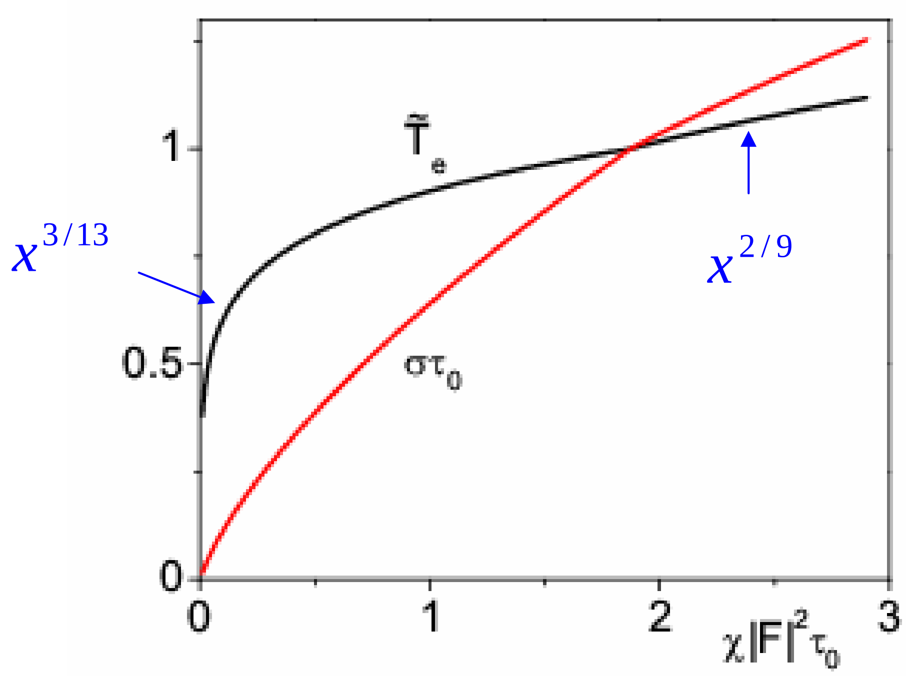
$$\tilde{T}_e = T_e/\omega_F$$

Absorption decrease: thermal population of the state $|2\rangle$ and bleaching

$$\sigma = \frac{|F|^2 \chi}{Z(T_e)}\left(1 - e^{-\omega_F / T_e}\right)$$

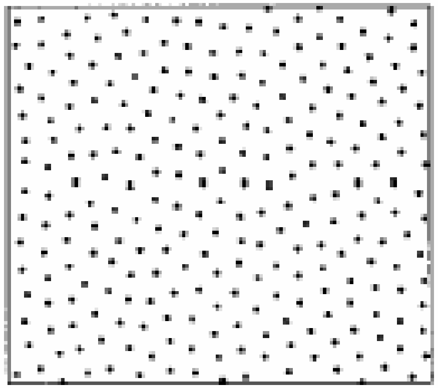
Kinematic two-ripplon scattering





$$\tilde{T}_e = T_e / \omega_F$$

Many-electron line shift



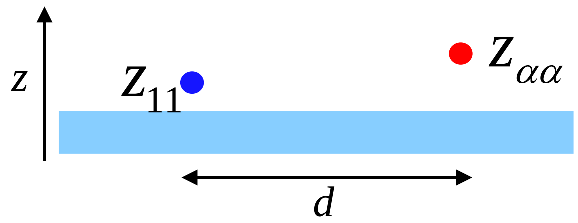
Strongly correlated electron liquid for $e^2(\pi n)^{1/2} >> T_e$

Different distance from He surface in different states leads to dependence of electron transition frequency on states of neighboring electrons:

$\rightarrow$ **many-electron Stark shift**

$$\varepsilon_2 - \varepsilon_1 \rightarrow \varepsilon_2 - \varepsilon_1 + \Omega(T_e)$$

"Many" nearest neighbors: mean-field approximation



$$H_{ee} \approx -\frac{e^2}{4}\sum_{n\neq m}(z_n - z_m)^2 / r_{nm}^3$$

$$\Omega(T_e) \approx \underbrace{\left\langle \sum_{m(\neq n)} e^2 r_{nm}^{-3} \right\rangle}_{\approx 8.9 e^2 n_s^{3/2}} \left[(z_{22} - z_{11}) \left(\sum_{\nu} z_{\nu\nu}\rho^{\nu\nu} - z_{11} \right) + |z_{12}|^2 \left(\rho^{11} - \rho^{22} - 1\right) \right]$$

Energy balance equation

$$\frac{T_e - T}{\omega_F} = \frac{|F|^2 \chi\tau}{Z(T_e)}\left(1 - e^{-\omega_F/T_e}\right)$$

with

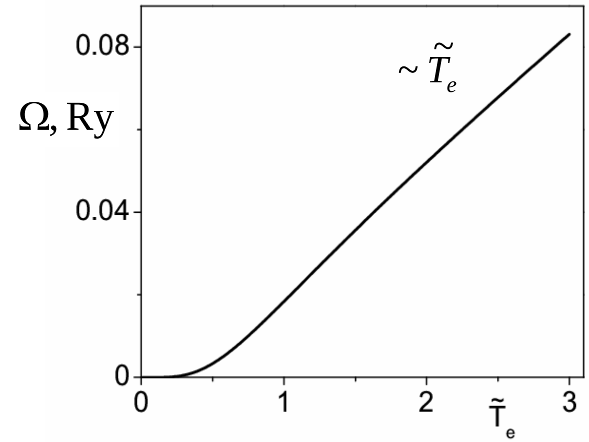
$$\chi = \chi(T_e) = \frac{\Gamma_0}{\Gamma_0^2 + [\delta\omega - \Omega(T_e)]^2}$$

Self-consistent equation for ee temperature

$$\frac{\tilde{T}_e Z(\tilde{T}_e)}{1-e^{-1/\tilde{T}_e}} = |F|^2 \chi(\tilde{T}_e)\tau, \quad \tilde{T}_e = \frac{T_e}{\omega_F}$$

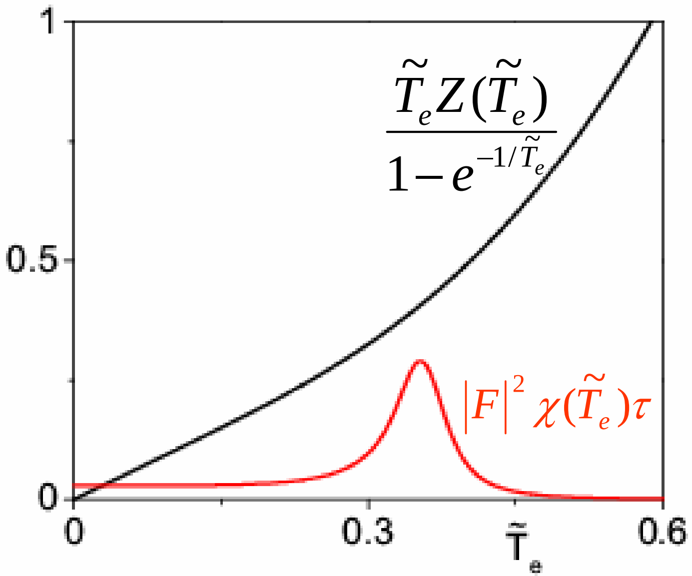
$\chi(\tilde{T}_e)$ has a narrow peak for $\delta\omega = \Omega(\tilde{T}_e^*)$ if

$$[d\Omega/d\tilde{T}_e]\Big|_{\tilde{T}_e^*} \gg \Gamma_0$$

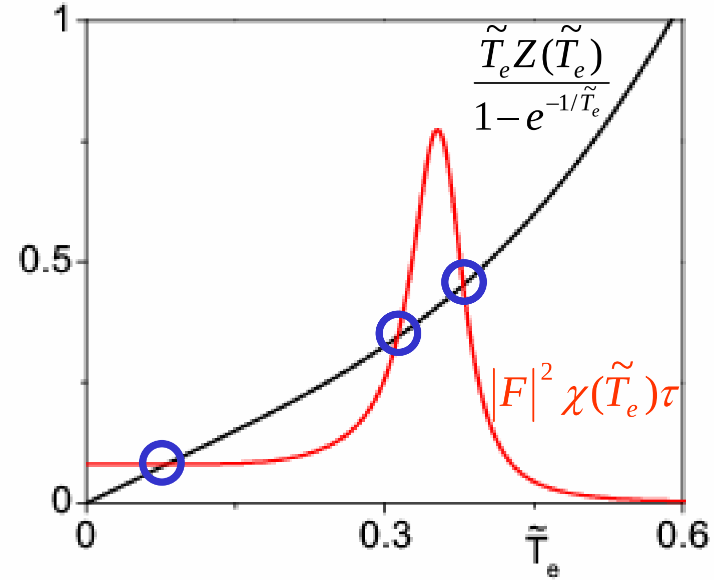


Constant energy relaxation rate approximation

weak field



strong field → bistability



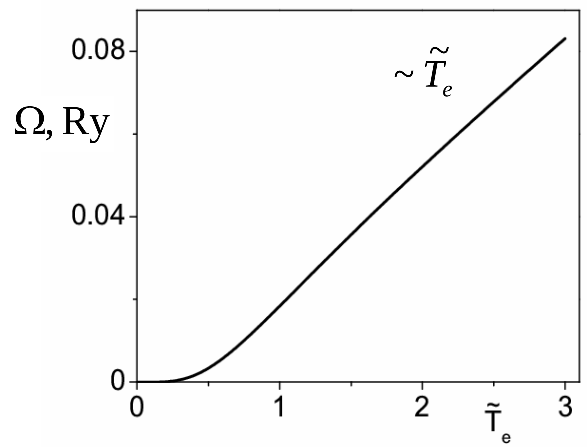
$$\delta\omega/\Gamma_0 = 1, \delta\omega = 0.001\text{Ry}$$

Self-consistent equation for ee temperature

$$\frac{\tilde{T}_e Z(\tilde{T}_e)}{1-e^{-1/\tilde{T}_e}} = |F|^2 \chi(\tilde{T}_e)\tau, \; \tilde{T}_e = \frac{T_e}{\omega_F}$$

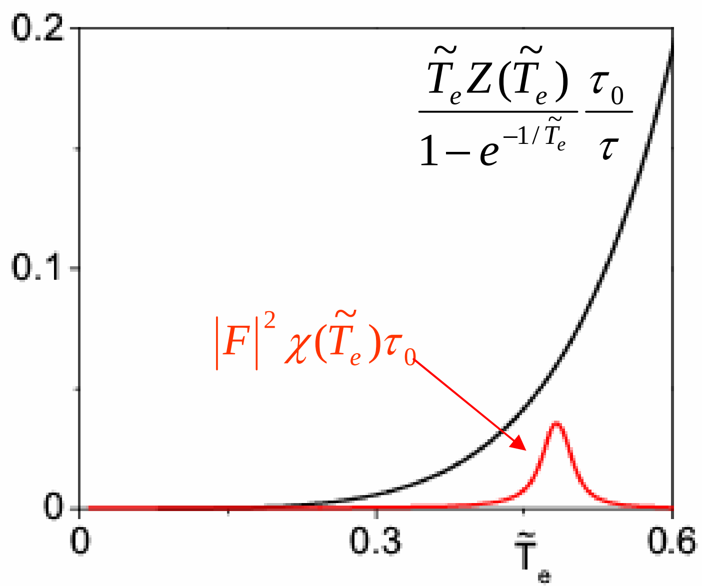
$\chi(\tilde{T}_e)$ has a narrow peak for $\delta\omega = \Omega(\tilde{T}_e^*)$ if

$$[d\Omega/d\tilde{T}_e]\Big|_{\tilde{T}_e^*} >> \Gamma_0$$

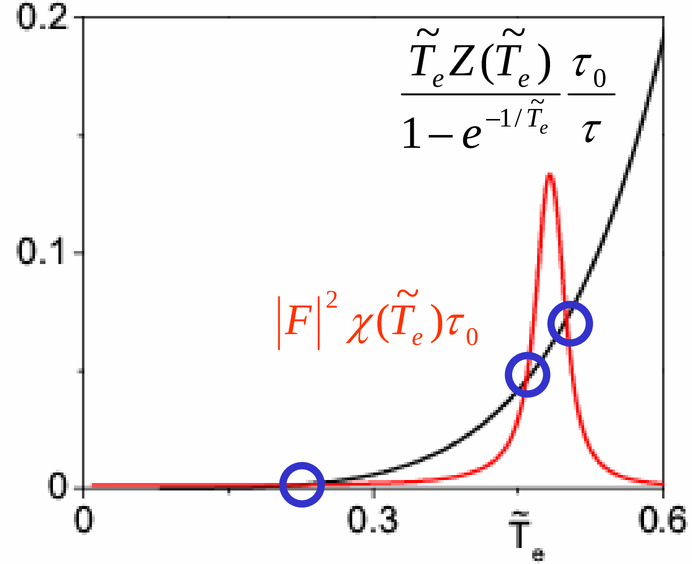


Kinematic two-ripplon scattering

weak field



strong field → bistability



$$\delta\omega/\Gamma_0 = 10, \delta\omega = 0.003\text{Ry}$$

- **Absorption saturation is accompanied by bleaching from electron heating.**
- **Many-electron shift of transition frequency leads to absorption hysteresis for low helium temperatures**
- **Electron energy relaxation can be studied via nonlinear absorption experiments**